A Missile Duel Between Two Aircraft

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A delicate differential game is formed by two aircraft performing a long-range missile duel. The duel starts for each aircraft with a semiagressive phase until the launch of the missile, when a pure evasion commences. In numerically solving the nonlinear differential game problem, an optimization algorithm is used consisting of a modified first-order differential dynamic programming method combined with an effective convergence-control technique. The three-dimensional spatial motion of point mass vehicles with realistic models is treated. The results of optimizing the problem point out the potential of new tactics. If the vehicles are free to maneuver in altitude, the contributions from the missiles will dominate the outcome of the game. Of particular importance is the desirable ability to enter the game at a higher altitude than that of the opponent.

I. Introduction

TWO aircraft, initially separated at a very long distance, are assumed to perform a long-range missile duel. Each aircraft starts with an aggressive phase in searching for a good launch position for its missile, simultaneously avoiding a poor evasive position. At the launch moment, the aircraft enter a pure evasive phase. See Fig. 1. This problem places demands on the mixture of turning and speed performance of the aircraft and the range capability of the missiles. An optimal solution of the problem points out how to make use of the full performance of the participating vehicles. Also, a particular goal of the study is to investigate a standoff tactic.

In maneuvering, the vehicles are free to use changes in altitude, heading, and longitudinal acceleration. We assume a missile with full "launch-and-forget" capability. The models for the aircraft and missiles are fully realistic, including such practical constraints as limits on load factor, dynamic pressure, and altitude. The trajectories of the aircraft as well as the missiles are optimized. This complex optimization problem is solved by a modified first-order differential dynamic programming (DDP) method as described in Ref. 1. It is extended to cover the transversality condition for the optimal control problem due to the launch mechanism.

Related works have recently been presented by Well[2] and Walden.[3] They replace the optimization of the missiles' controls by using proportional navigation, given a realistic model of today's missiles. However, this approach gives a short range to the missiles. In most cases, the essential contribution to the optimization originates from the altitude profile of the missiles, as will be shown in this paper. Well[2] also discusses using missiles without complete launch-and-forget capability, forcing the aircraft not to turn beyond the gimbal limit. The cost criterion is modeled very similarly in Refs. 2-5 and here. The numerical methods in Refs. 2 and 3 are of second order. Well[2] suggests a multiple-shooting method, which is fast in convergence when close to the optimal solution. The method is tricky to handle due to the sensitivity to the starting condition of the algorithm. Walden[3] uses a nonlinear programming method based on a quasi-Newton algorithm. This method is robust but is computationally demanding.

The optimal solution obtained from whichever numerical method is used is an open-loop control. In practice, we want to have a closed-loop control. However, such a closed-loop control is impossible to obtain for this very nonlinear problem. Nevertheless, the solutions obtained are very valuable. For example, we obtain hints as to how to form practical tactics, design practical control laws, and trade off the turning and speed capabilities of the aircraft.

The purpose of this paper is to formulate the duel problem as a zero-sum, two-person differential game with perfect information. The solutions will show new results and might be the basis for developing new tactics.

II. Dynamic Equations

A three-dimensional spatial motion of a point-mass vehicle, as in Fig. 1 and the Appendix, is described by

$$\dot{x} = v\cos\gamma\cos\chi/1000 \tag{1}$$

$$\dot{y} = v\cos\gamma\sin\chi/1000 \tag{2}$$

$$\dot{h} = v\sin\gamma/1000 \tag{3}$$

$$\dot{\gamma} = (u_1 - \cos\gamma)g/v \tag{4}$$

$$\dot{\chi} = u_2 g/(v\cos\gamma) \tag{5}$$

$$\dot{v} = u_3 T/m - D_0/m - (u_1^2 + u_2^2)D_i/m - g\sin\gamma \tag{6}$$

where the dividing by 1000 gives the horizontal coordinates x and y and the altitude h in kilometers; and γ is the climbing angle, χ the course angle relative to the x axis, v the speed, and g the acceleration due to gravity. The thrust deflection due to the angle of attack has been omitted. The load factor and bank angle have been replaced by a vertical control u_1 and a horizontal control u_2 subject to the load factor constraints [see Eqs. (A7) and (A8) in the Appendix]. The control $u_3 \in [0,1]$ is the throttle setting (which is applicable for the aircraft only). The thrust T and aerodynamic drag terms D_0 and D_i are detailed in the Appendix. The mass m is assumed constant for the aircraft.

The complete problem consists of four vehicles, each subject to a set of differential equations such as Eqs. (1-6) and constraints as noted in the Appendix. This gives a total of 24 differential equations with 10 control variables. By shortening Eqs. (1-6) for each vehicle, the complete dynamic system can be written as

$$\dot{x}_{A1} = f_{A1}(x_{A1}, u_{A1}; t), \qquad x_{0_{A1}} = x_{A1}(t_0) \tag{7}$$

$$\dot{x}_{M1} = f_{M1}(x_{M1}, u_{M1}; t), \qquad t_{l_1} \le t, \; x_{0_{M1}} = x_{A1}(t_{l_1}) \tag{8}$$

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$$\dot{x}_{A2} = f_{A2}(x_{A2}, u_{A2}; t), \qquad x_{0_{A2}} = x_{A2}(t_0) \tag{9}$$

$$\dot{x}_{M2} = f_{M2}(x_{M2}, u_{M2}; t), \qquad t_{l_2} \leq t, \qquad x_{0_{M2}} = x_{A2}(t_{l_2}) \tag{10}$$

where $x_{(\cdot)}$ is the state vector (x,y,h,γ,χ,v) and $u_{(\cdot)}$ the control vector (u_1,u_2,u_3). The indices A1, A2, M1, and M2 refer to the corresponding vehicles in Fig. 1. The f_{A1} and f_{A2} change at the launch time due to the weight and drag losses associated with the missile. These effects are considered in the examples run, but do not operate on the theory discussed herein and are therefore omitted in subsequent discussion.

III. Optimization Problem

Since the initial range is large, the best trajectory for either aircraft will end with a missile pursuing in almost a tail chase, see Fig. 1. This pursuit-evasion game belongs to a particular class of last-ditch maneuver or side-stepping games, related to the study in Ref. 6. Reference 7 is representative of other related papers that can be found in the literature. This end game might involve particular differential game eccentricities affecting the optimization of the previous part of the game. Additional convergence difficulties, probably insurmountable, will then occur there. Also, when one missile obtains a hit, there is no information regarding the hit margin that should be utilized by the other aircraft. For these reasons, it is necessary to separate the long-range duel from the two end games. From studies of the end game, it is seen that the hit capability is strongly related to the speed advantages the missile has when closing in on the opposing aircraft. This speed advantage is then sought to be maximized by the missile and minimized by the aircraft. However, it is more practical to convert this measure to optimize the distance at a specified closing velocity v_{cl}. A zero-sum differential game of degree can be formulated by the performance criterion,

$$V(x_0,t_0) = \underset{A_1,M_1}{\text{Min}} \ \underset{A_2,M_2}{\text{Max}} \ [R_1(t_{f_1}) - R_2(t_{f_2})] \tag{11}$$

where x_0 and t_0 represent the initial conditions. The final times t_{f_i}, $i=1,2$, are, for practical reasons, assumed to be sufficiently larger than the launch times t_{l_i} and are defined as

$$t_{f_i} = \underset{t}{\arg}[\dot{R}_i(t) = -v_{cl}] \tag{12}$$

According to the above discussion, the initial range must be chosen large enough to give the smallest R_i a value of some kilometers. In the optimal solution obtained, the cost value [Eq. (11)] is a rough measure of how much the initial range can be shrunk to let either missile hit. In the examples below, v_{cl} in Eq. (12) is set equal to 0.025 km/s, corresponding to the closing velocity the missiles need in order to hit nonmaneuvering aircraft.

The object is to find the differential game saddle point for Eq. (11) subject to Eqs. (7-10) and the constraints described in the Appendix. Assume the value of the game [V in Eq. (11)] exists, and introduce adjoint vectors for each vehicle, $V_{x_{A1}}$, $V_{x_{M1}}$, $V_{x_{A2}}$ and $V_{x_{M2}}$, each of dimension six. Concerning the particular problem stated, the dimensions in Eqs. (15-18) of the vectors $V_{x_{A1}},\ldots, V_{x_{M2}}$ can be reduced to three. The Hamiltonian will then be,

$$\begin{aligned} H = {} & V'_{x_{A1}} f_{A1}(x_{A1},u_{A1};t) + V'_{x_{M1}} f_{M1}(x_{M1},u_{M1};t) \\ & + V'_{x_{A2}} f_{A2}(x_{A2},u_{A2};t) + V'_{x_{M2}} f_{M2}(x_{M2},u_{M2};t) \\ & + \psi_{A1}(x_{A1},u_{A1};t) + \psi_{M1}(x_{M1},u_{M1};t) \\ & + \psi_{A2}(x_{A2},u_{A2};t) + \psi_{M2}(x_{M2},u_{M2};t) \end{aligned} \tag{13}$$

The prime (′) stands for transpose and $\psi_1,\ldots,\psi_4$ are nonlinear functions representing the constraints on the controls as in Fig. A7. This can be implemented by the well-known multiplier method. The details are omitted in this paper, but they can be found in Ref. 1. A necessary condition for a differential game saddle point is a min-max of the separable Hamiltonian equation (13), subject to the given constraints. This gives the optimal controls,

$$(u^{*\prime}_{A1}, u^{*\prime}_{M1}, u^{*\prime}_{A2}, u^{*\prime}_{M2}) = \arg \underset{u_{A1},u_{M1}}{\text{Min}} \ \underset{u_{A2},u_{M2}}{\text{Max}} \ \{H\} \tag{14}$$

The adjoint vectors must satisfy the adjoint equations

$$\dot{V}_{x_{A1}} = -H_{x_{A1}} \tag{15}$$

$$\dot{V}_{x_{M1}} = -H_{x_{M1}}, \qquad t_{l_1} \leq t \tag{16}$$

$$\dot{V}_{x_{A2}} = -H_{x_{A2}} \tag{17}$$

$$\dot{V}_{x_{M2}} = -H_{x_{M2}}, \qquad t_{l_2} \leq t \tag{18}$$

The adjoint variables must satisfy certain transversality conditions. The end points condition at t_{f_1} and t_{f_2} will be derived from

$$R_2 = \| (x,y,z)_{A1} - (x,y,z)_{M2} \| \tag{19}$$

Then

$$R_{2_{x_{A1}}} = -R_{2_{x_{M2}}} \tag{20}$$

This gives at t_{f_2}

$$V_{x_{A1}}(t_{f_2}) = -V_{x_{M2}}(t_{f_2}) = -R_{2_{x_{A1}}} \tag{21}$$

In the same way for A2 and M1,

$$V_{x_{A2}}(t_{f_1}) = -V_{x_{M1}}(t_{f_1}) = R_{1_{x_{A2}}} \tag{22}$$

We also have

$$V_{x_{A1}}(t) = V_{x_{M2}}(t) = 0, \qquad t > t_{f_2} \tag{23}$$

$$V_{x_{A2}}(t) = V_{x_{M1}}(t) = 0, \qquad t > t_{f_1} \tag{24}$$

Finally, we have a particular condition at t_{l_1} and t_{l_2} from which the new type of optimality conditions at the launch moments can be derived (see Refs. 4 and 5) as

$$V_{x_{A1}}(t_{l_1}^-) = V_{x_{A1}}(t_{l_1}^+) + V_{x_{M1}}(t_{l_1}^+) \tag{25}$$

$$V_{x_{A2}}(t_{l_2}^-) = V_{x_{A2}}(t_{l_2}^+) + V_{x_{M2}}(t_{l_2}^+) \tag{26}$$

By satisfying Eqs. (7-18) and (21-26), we have a solution that is a candidate for a differential game saddle point solution. This is the best we can do.[4,5] However, those equations are not easily solved. An excellent way to solve such an optimal control problem numerically is by using the differential dynamic programming (DDP) method.

Numerical Optimization Method

The background of the DDP method is decribed in Ref. 1; a short description will be given here.

The iterative computational procedure is initiated by assuming nominal control histories $\bar{u}_{A1}(t)$, $\bar{u}_{M1}(t)$, $\bar{u}_{A2}(t)$, and $\bar{u}_{M2}(t)$ subject to the control constraints, generating the corresponding nominal trajectories $\bar{x}_{A1}(t)$, $\bar{x}_{M1}(t)$, $\bar{x}_{A2}(t)$, and $\bar{x}_{M2}(t)$ by integrating Eqs. (7-10) forward in time from t_0 until Eq. (12) is satisfied, which determines t_{f_1} and t_{f_2}. The nominal controls and trajectories as well as the calculated cost value are stored.

The end-point boundary equations (21) and (22) are now known and the adjoint Eqs. (15-18) can be integrated backward in time along the nominal trajectories, while solving the optimal controls from Eq. (14). These new control histories must be stored. In parallel with integrating the adjoints, a predicted cost change will be calculated by integrating Eq. (28), $i=1,\ldots,10$ (dim $u=10$). Assume that the Hamiltonian equation (13) is separable in the components of each control variable,

$$H=\sum_{i=1}^{10} H^i(\bar{x},u_i,V_x;t) \tag{27}$$

where $\bar{x}$ and V_x represent the state and adjoint vectors, respectively, for the individual vehicle. The differential equation for the predicted cost change will then be

$$\dot{a}_i(t)=H^i(\bar{x},\bar{u}_i,V_x;t)-H^i(\bar{x},u_i^*,V_x;t)$$

$$a_i(t_{f_j})=0, \qquad j=1,2 \tag{28}$$

The predicted cost change is obtained at t_0 as

$$a(t_0)=\sum_{i=1}^{10} a_i(t_0) \tag{29}$$

Apply the new control histories and integrate Eqs. (7-10), subject to the constraints, forward in time-generating trajectories (which should be better than the nominal ones). Also calculate the new cost value and form the cost changes ΔV by subtracting the nominal cost value from the new one obtained. Comparing $a(t_0)$ with ΔV enables comprehension of the convergence status of the iteration just fulfilled. If the status is not satisfactory, which is very common, a convergence control technique must be applied.

Convergence Control

The reasons for bad convergence are excessively large changes in the controls. These changes can be restricted by introducing penalty terms in the Hamiltonian equation (27), which then are used in Eq. (14). The modified Hamiltonian will then be

$$\mathcal{H}=\sum_{i=1}^{10}\left[H^i(\bar{x},u_i^*,V_x;t)\pm C_i(\bar{u}_i-u_i^*)^2\right] \tag{30}$$

where the + stands for minimization and − for maximization and C_i is a convergence control parameter (CCP). This technique means that the contribution from a particular u_i^* is strongly related to the cost change by $a_i(t_0)$. Accordingly the effects of a certain control variable can be observed and the corresponding C_i can be used to prevent divergence or to speed up the convergence due to this particular control variable. It is very important to have this device, especially when the number of control variables is large. Otherwise, it would be very difficult to obtain practical acceptable convergence of such an iterative procedure. Use of the CCP technique is described in Ref. 1.

IV. Applications

The optimization algorithm can be used in searching for answers to several questions appearing in the missile duel stated. The results obtained in Ref. 5 point out what is important among the control steps, game parameters, aircraft performance parameters, etc. Three examples aimed at enlarging the understanding of the process will be given here. The initial situation used is the commonly occurring head-on encounter at a given distance R_0, which is chosen large in order to obtain a meaningful and consistent optimization (see the introductory remarks in Sec. III). However, in most cases, particularly at constant altitude, R_0 can be roughly decreased afterward in the solution until one of the missiles reaches the hostile aircraft. Then the magnitude of the cost value approximates the safety distance the other aircraft has with respect to the opponents' missile. A stationary solution obtained by the method previously described is not guaranteed to be the differential game solution to the problem at hand.[4,5] However, the results shown are assumed to be the correct differential game saddle point solutions. The model for the aircraft has the performance represented in Figs. A4 and A5 in the Appendix. Modeled as long range and without restrictions on the launch conditions, the missiles are assumed to have full launch-and-forget capability. The launch times are assumed fixed and set to 10 s. Corresponding vehicles on both sides are equal if not otherwise stated.

Example 1

Playing with the variable altitude has a strong effect on the result. In order to isolate the influence of a certain parameter, the altitudes for all of the vehicles have to be fixed. One trivial case is plotted in Fig. 2. In this case, if we shorten the initial distance by about 15.1 km, M2 will hit but M1 will miss by about 700 m. The control histories for this case are found in Fig. 3. The effect of the launch mechanism [Eqs. (25) and (26)] is clearly seen here, where the aircraft decreases the turning before launch and switches to full turning at launch time. This is performed as an appropriate balance, giving the missile

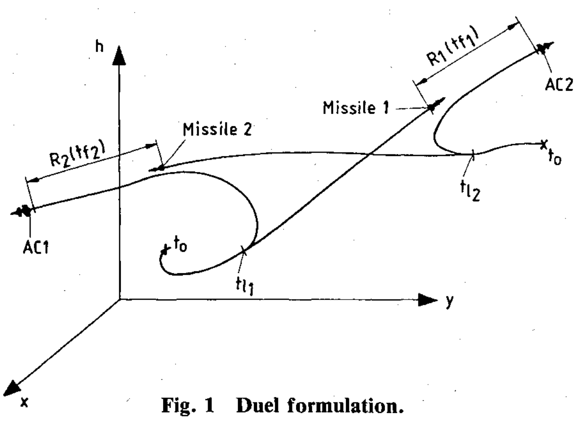

Fig. 1 Duel formulation.

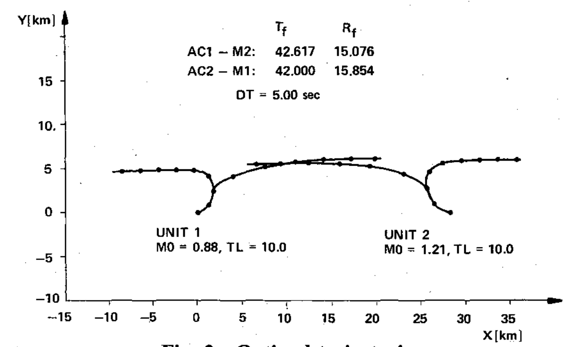

Fig. 2 Optimal trajectories.

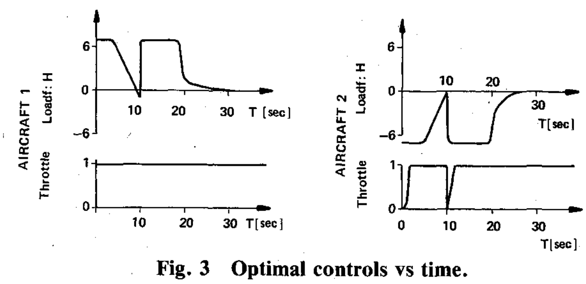

Fig. 3 Optimal controls vs time.

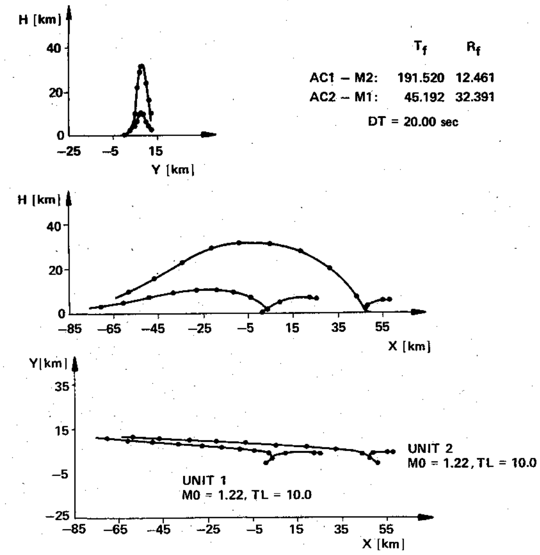

Fig. 4 Optimal trajectories, 50% sustained thrust reduction of M1.

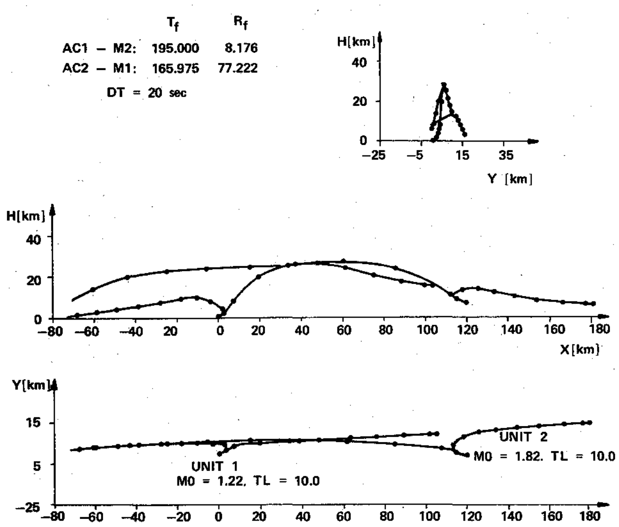

Fig. 5 Optimal trajectories, $\Delta h_0 = 6.6$ km.

a good launch position in parallel to having a preferable evasive position. The balance is accomplished in accordance with the performance criterion stated by Eq. (11), which here means that both players value as much a kill of the opponent's aircraft as a loss of their own. This effect also gives an interesting picture of the throttle control for the aircraft with the higher initial velocity. Having the throttle off will improve the evasive turning. However, these small time intervals with the throttle off do not affect the results substantially (in the region of 100 m); besides, it is not practically feasible to do this.

Example 2

In this case, the sustained thrust phase of missile M1 is reduced by 50% compared to the properties of the reference missile otherwise used. Also, the launch mass of the missile is reduced by the lower powder mass. The optimal trajectories are plotted in Fig. 4. There is no longer a throttle-off period due to the need for a hard pull up at the beginning. The main purpose of the initial climb seems to be to achieve a proper launch angle in attaining high altitudes for the missiles. The maximum altitudes reached are about 31 km for M2, 7.6 km for M1, 11 km for A1, and 5.8 km for A2. The advantage for unit 2 is considerable —20 km.

Example 3

Both units have now equal components. The principal difference is in the initial altitudes, which for unit 1 is sea level as before, whereas unit 2 starts at 6.6 km with a higher initial velocity. This results in a tremendous gain in the cost value (69.0 km), to the advantage of unit 2. The initial velocity and altitude can be converted to the altitude-energy equivalences, which for units 1 and 2 are $E_1 = 8.76$ km and $E_2 = 16.73 + 6.6 = 23.33$ km, respectively. The difference is a minor part of the cost value. The important contributions are the higher launch velocity and the thinner air as start conditions for M2 (6.6 km corresponds to half the air density relative to that at sea level). The maximum altitudes during the flight are about 28 km for the missiles, 10 km for A1 and 14 km for A2. This problem is very difficult to solve and the trajectories obtained, shown in Fig. 5, might differ slightly from the optimal ones. The cost value might be within a kilometer in accuracy.

From these two cases we learn that the tactics in choosing the initial energy, particularly the altitude, are very important. The tactics can well compensate for a weaker component. A variable altitude has a considerable influence on the outcome. The altitude variations for the missiles are considerable. This amount of altitude variation might not be practical due to the lower controllability at high altitudes radar tracking at long distances, etc., Thus, the gain in using altitude variations ought to stimulate the improvement of technology and tactics.

V. Summary

Realistic long-range combat has been modeled as a differential game including real models and constraints of aircraft and missiles. The models are of a high-performance type; hence, the results might be somewhat unusual, including behavior such as rather large altitude variations. But the results point out important features of the combat problem and attributes of the aircraft/missile system. In the measures of the game, there are relatively large "miss distances" of several kilometers involved, which might be confusing. However, the value of the game is deemed to be a representative measure of the advantage one aircraft has over another.

The stated differential game comprises a very complex optimization problem. In solving this problem, an optimization algorithm based on a first-order differential dynamic programming (DDP) method is used. The method has been extended to cover the launch process. The success in obtaining a solution from such an iterative algorithm is that an effective convergence control technique is a must.

The largest contribution to the outcome must be credited to the missiles' altitude profiles. A missile's deceleration is very drag dependent when the rocket engine is shut down. Therefore, the missile goes a long distance at a high speed if the air density is low. In the duel formulation stated, the tactics should be to launch the missile at a closing distance of about 40 km at sea level or up to 95 km at an altitude of 6.6 km. Then the missile must be able to operate at altitudes around 30 km along the greater part of the flight path.

Appendix

The models for aircraft as well as for missiles are based on nonlinear functions derived from the theory of aerodynamics and wind tunnel tests. These functions have been modeled by fitting polynomials and known functions in order to attain representative derivatives without artificial oscillations. The longitudinal acceleration of an aircraft or a missile, neglecting the thrust deflection due to the angle of attack, is in general given as

$$\dot{v} = [T(M,h) - D(M,h,n)]/m - g\sin\gamma \tag{A1}$$

The drag consists of a zero-lift component and an induced component,

$$D = qS[C_{D_0} + (C_{D_i}/C_L^2)C_L^2] = D_0 + D_i(u_1^2 + u_2^2) \tag{A2}$$

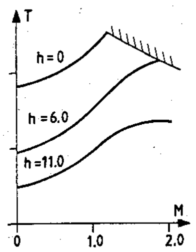

Fig. A1 Thrust vs Mach number and h.

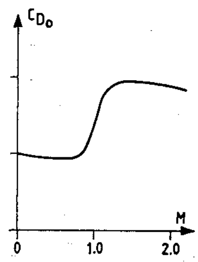

Fig. A2 Zero-drag coefficient vs Mach number.

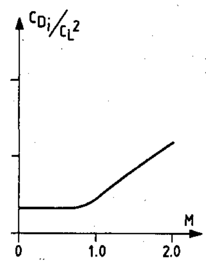

Fig. A3 Induced coefficient vs Mach number.

where S is the wing area in square meters, C_{D_0} the zero-lift drag coefficient, C_{D_i} the induced drag coefficient, C_L the lift component, and q the dynamic pressure

$$q = \tfrac{1}{2}\rho(h)v^2 \tag{A3}$$

where $\rho(h)$ is the air density, related to the ICAO standard atmosphere.

The relation between the load factor n and C_L is,

$$C_L = nmg/Sq \tag{A4}$$

Inserted into Eq. (A2), this gives an expression for Eq. (A1) as

$$\dot{v} = \frac{T}{m} - C_{D_0}\frac{Sq}{m} - \left(\frac{C_{D_i}}{C_L^2}\right)\frac{mg^2}{Sq}n^2 - g\sin\gamma \tag{A5}$$

Dependencies of T, C_{D_0}, and (C_{D_i}/C_L^2) on Mach number are represented in Figs. A1-A3.

The performance of an aircraft is well represented by the specific excess power (SEP), defined as

$$\text{SEP} = \frac{T-D}{mg}v \tag{A6}$$

The aircraft model used has a mass of 9000 kg and a reference wing area of 30 m^2. Typical SEP plots are shown in Figs. A4 and A5.

The aerodynamics of the missiles are in large part similar to the aerodynamics of the aircraft. Plots corresponding to Figs. A2 and A3 were used in modeling the missile. On the other hand, the thrust is different; the rocket thrust has a boost and a sustained phase in accordance with Fig. A6. The mass of each missile varies with time in a similar profile, from an initial mass of 210 kg to the burnout mass of 125 kg. As a reference area, 0.031 m^2 was used.

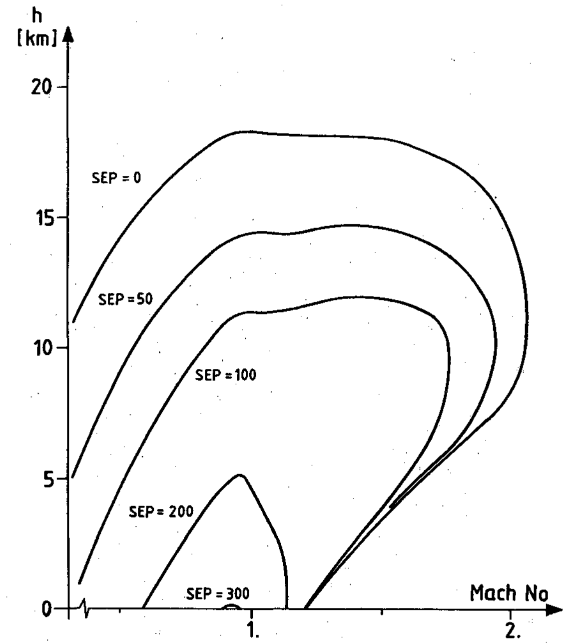

Fig. A4 SEP (m/s) vs Mach number and altitude ($n = 1$).

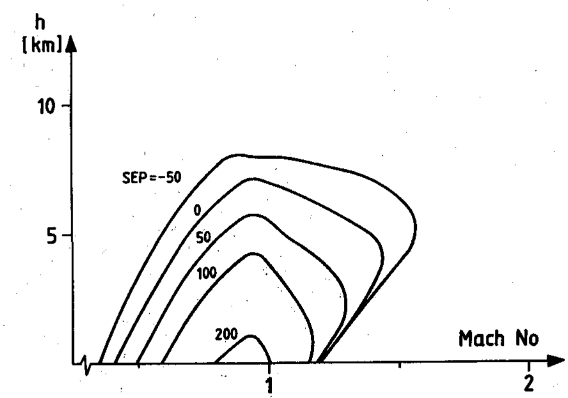

Fig. A5 SEP (m/s) vs Mach number and altitude ($n = 5$).

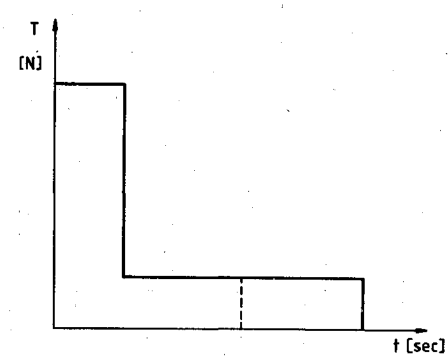

Fig. A6 Thrust vs time.

Constraints

The maximum load factor of each aircraft or missile is constrained by the maximum lift coefficient at speeds below the corner speed v_c or by a structural limit at higher speeds. The maximum load factor varies with the velocity as shown in Fig. A7.

The constraint is here formed as,

$$|n| \leq \hat{n}(v/v_c)^2 \quad \text{if} \quad v \leq v_c \tag{A7}$$

$$|n| \leq \hat{n} \quad \text{if} \quad v \geq v_c \tag{A8}$$

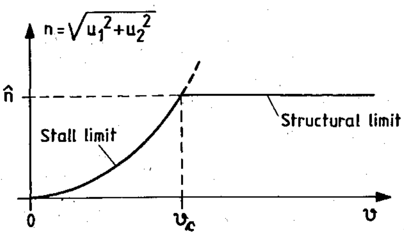

Fig. A7 Constraint of the control.

The aircraft are in addition constrained to $h \geq h_{min}$ and $q \leq q_{max}$, where q denotes the dynamic pressure. The limit values are chosen to be $h_{min} = 100$ m, with q_{max} usually corresponding to $M = 1.2$ at $h = 0$. These constraints are modeled in the physical equations by adding terms in Eqs. (4) and (6) in such a way that the system can not override these state constraints (see Ref. 5).

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